MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

28 MAY 2019
2.30 P.M. – 4.30 P.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only excluding the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE (5)** questions.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.
5. Formulas are provided and attached in Appendix.

Question 1

a. Given the points $P(1, 2, -1)$, $Q(2, 3, 1)$ and $R(3, -1, 2)$,

i. find the vectors $\vec{PQ}$ and $\vec{PR}$. **[1 mark]**

ii. find the cross product of $\vec{PQ} \times \vec{PR}$. **[2.5 marks]**

iii. determine the equation of the plane that contains the points P, Q and R. **[1.5 marks]**

b. Given the system of linear equations as follows,

$$\begin{aligned} 2x_1 + 5x_2 - x_3 &= 4 \\ x_1 + 3x_2 &= 8 \\ -2x_1 + 2x_2 + 4x_3 &= 12 \end{aligned}$$

i. write the system into a matrix equation form, $AX = B$. **[1 mark]**

If the cofactor of A is $C = \begin{bmatrix} 12 & -4 & 8 \\ -22 & 6 & -14 \\ 3 & -1 & 1 \end{bmatrix}$,

ii. find the determinant of A. **[1 mark]**

iii. solve the system using the inverse method. **[3 marks]**

Question 2

a. Given the complex numbers $z = 3+3i$ and $w = 3+5i$,

i. write z and w in polar form. **[5 marks]**

ii. write z and w in exponential form. **[1 mark]**

b. Calculate $\left\{2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right\}^3$. **[2 marks]**

c. Simplify $\dfrac{(\cos\theta + i\sin\theta)^4(\cos 3\theta + i\sin 3\theta)^2}{\left(\cos\frac{1}{3}\theta + i\sin\frac{1}{3}\theta\right)^3}$. **[2 marks]**

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Question 3

a. Find the limit.

i. $\lim_{x\to 1} \sqrt{4x^4+x^3-x}$. **[1 mark]**

ii. $\lim_{x\to -\infty} \frac{2x^4-3x-9}{5x^8-3x^3+0.5}$ by using L'Hospital rule or other method. **[1.5 marks]**

b. Find the derivative of the following functions.

i. $y=\frac{5x}{2x^3-2x}$ by using quotient rule or other method. **[2 marks]**

ii. $y=\left(x^4+0.4x-5\right)^3$ by using chain rule or other method. **[2 marks]**

c. Integrate

i. $\int_0^2 (3x^2+\cos x-4)\ dx$. **[1.5 marks]**

ii. $\int -2x^3e^x\ dx$ by using integration by part. **[2 marks]**

Question 4

a. The first order differential equation is given below:

$$\frac{dy}{dx}=\frac{2\sin x}{5+2y^3-3y}$$

Solve the first order differential equation using separable method. **[2 marks]**

b. Find the general solution of the following homogeneous linear second order differential equation.

$$2y''-y'-y=0$$ **[3 marks]**

c. Determine whether the given differential equation, $(-3x+3y)dx+(3x+4y^2)dy=0$, is **exact.** Then, solve it. **[5 marks]**

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Question 5

A secondary school teacher wants to determine whether the number of hours his students spent on smartphone activities (such as browsing internet, social media, smartphone applications, etc.) during weekends had any effect on the final exam result of his subject. The teacher investigates 8 random students and the following data was recorded.

Time spent on smartphone's activities (Hours), x	5	2	12	9	15	6	25	16
Final Exam Result, y	64	87	50	71	44	56	42	60

a. Find $\sum x$, $\sum y$, $\sum x^2$, $\sum y^2$ and $\sum xy$. **[2.5 marks]**

b. Find a regression model to fit the data. **[4 marks]**

c. Using the regression model from part (b), what will a student score be for his final exam if he spent 10 hours on smart phone activities? **[1 mark]**

d. Calculate the correlation coefficient, r. Then, explain the relationship between the variables. **[2.5 marks]**

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APPENDIX

1. Equation of plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
2. Characteristics equation: $|\lambda I - A| = 0$
3. Eigenvectors: $(\lambda \mathrm{I} - A)X = 0$
4. Complex numbers:

   $r = \sqrt{x^2+y^2}$

   If x is positive, $\theta = \tan^{-1}\frac{y}{x}$

   If x is negative, $\theta = \tan^{-1}\frac{y}{x} + \pi$

   Polar form: $z = r(\cos\theta + i\sin\theta)$

   Exponential form: $z = re^{i\theta}$

   De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

5. Criterion for an Exact Differential Equation : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
6. General solution of the homogeneous linear second order differential equation

   For Distinct real roots: $y = c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x}$

   For Repeated roots: $y = c_1e^{\lambda_1 x} + c_2xe^{\lambda_2 x}$

   For Complex conjugate roots: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

7. Derivative Formulas

$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}\ln x = \frac{1}{x}$
Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\frac{d}{dx}(f \cdot g) = f\frac{dg}{dx} + g\frac{df}{dx}$
Quotient Rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$ with $g(x) \neq 0$
Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

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8. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
$\int \frac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$
$\int \cos kx dx = \frac{\sin kx}{k} + C$
Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then $\int f(u)du = \int f(g(x))\frac{d}{dx}g(x)dx$
Integration by Part: $\int udv = uv - \int vdu$

9. $\bar{x} = \frac{\sum x}{n}$

$$S_{xy} = \sum xy - \frac{\left(\sum x \sum y\right)}{n}, \qquad S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}, \qquad S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

10. $\beta_1 = \frac{S_{xy}}{S_{xx}}$, $\beta_0 = \bar{y} - \beta_1 \bar{x}$

11. Least square regression line: $\hat{y} = \beta_0 + \beta_1 x$

12. Sample correlation coefficient = $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

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